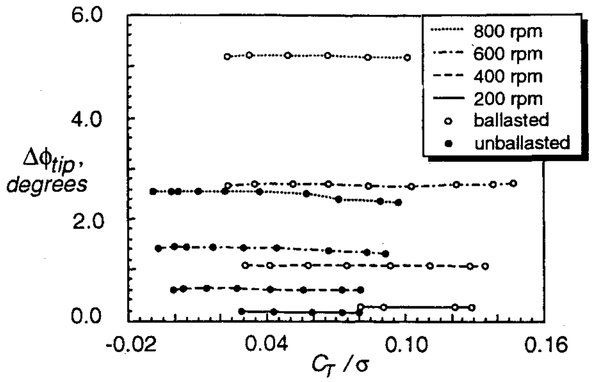

Fig. 3 Change in blade elastic twist at tip as a function of thrust coefficient-solidity ratio.

Discussion of Results

A set of extension-twist-coupled composite model rotor blades was tested as previously described. The change in blade elastic twist, $\Delta\phi_{\text{tip}}$ (positive nose up) is plotted as a function of rotor speed for test and analysis, using both mass distributions, in Fig. 2. The data points represent twist values recorded for the lowest available collective pitch settings at each rpm interval. Test results showed maximum twists of 2.54 deg for the unballasted blade configuration and 5.24 deg for the ballasted blade. These results were compared with a geometrically nonlinear finite element analysis that yielded maximum twists of 3.02 deg for the unballasted blade configuration and 5.61 deg for the ballasted blade configuration.

The effect of blade collective pitch on the blade elastic twist is shown in Fig. 3 for both blade configurations. The change in blade elastic twist is plotted as a function of the thrust coefficient-solidity ratio C_T/σ. Typically, an increase in collective blade pitch results in increased thrust, which causes the blades to flap upward. The magnitude of this flap angle, if significant, could decrease the effective centrifugal force acting on the blade, thereby decreasing the magnitude of twist obtained. However, no appreciable degradation in the blade elastic twist was observed for either configuration because the blade coning angle remained small throughout the sweep, reaching a maximum of only 2.5 deg during the ballasted configuration test run.

Although the total change in twist achieved in this preliminary study (5.24 deg over 0–100% rotor speed) indicates that passive blade twist control is feasible, significantly larger twist changes are required for practical tilt rotor blade designs. As indicated in Ref. 3, this goal can be realistically achieved by decreasing torsional stiffness and including blade tip weight (features not exploited in this study) in an extension-twist-coupled blade design.

Conclusions

This study has demonstrated the feasibility of passive blade twist control for composite rotor blades. Hover testing of the set of blades produced maximum twist changes of 2.54 deg for the unballasted blade configuration and 5.24 deg for the ballasted blade configuration. These results compared well with those obtained from a detailed finite element analysis model of the rotor blade, which yielded maximum twists of 3.02 and 5.61 deg for the unballasted and ballasted blade configurations, respectively. The effect of collective pitch sweep on the blade elastic twist was also demonstrated to be minimal, since no appreciable degradation in twist was observed at increased collective pitch angles. Results from this study provide a basis for the design of a highly twisted extension-twist-coupled model blade for tilt rotor aircraft by demonstrating the potential for improved dynamic and aerodynamic characteristics.

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Contribution of the Truncated Modes to Eigenvector Derivatives

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I. Introduction

THE usefulness of modal sensitivities for analysis and design of structural systems is well known. Some specific applications include redesign of vibratory systems and design of control systems by pole placement. Therefore, methods of computing eigenvector derivatives have been an active area of research since the work of Fox and Kapoor.[1] Recent development in this area was surveyed by Haftka and Adelman.[2] Basically, there are two methods for computing eigenvector derivatives: algebraic methods and modal superposition methods. Algebraic methods using only the eigenvector of concern are available in Refs. 1 and 3. Since algebraic methods require special manipulation of the system matrix for each eigenvector of concern, they can not be implemented easily and are not efficient if a large number of eigenvector derivatives are required. To obtain exact solutions, Nelson's method[3] is the most efficient of the algebraic methods if few eigenvector derivatives are required. However, when many eigenvector derivatives are required, modal superposition methods are more efficient. Because of the cost of generating computer solutions for a dynamic analysis, it is often impractical to obtain all modes. Therefore, only the first low-frequency modes are computed and are used as basis vectors of eigenvector derivatives. However, modal truncation induces errors which can be significant if more high-frequency modes are truncated. An explicit method to improve the truncated modal superposition representation of eigenvector derivatives is presented in Ref. 4, in which a residual static mode is used to approximate the contribution due to unavailable high-frequency modes. The numerical performance of the cited methods is compared in Ref. 5.

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This Note presents a more accurate modal superposition method than Ref. 4. In this method, the contribution of the truncated modes to the eigenvector derivative is exactly expressed as a convergent series that can be evaluated by a simple iterative procedure. Because the contribution from the unavailable high-frequency modes is added to the eigenvector derivative, this method significantly improves the accuracy of the previously published method,[4] and it also provides error estimates for the computed results.

II. Analysis

A. Basic Equations for Eigenvector Derivatives

Consider the eigenvalue problem for undamped systems in structural dynamics,

$$Kx_i = \lambda_i M x_i \tag{1}$$

$$x_i^T M x_i = 1 \tag{2}$$

where K is the symmetric stiffness matrix, M the mass matrix, λ_i the ith eigenvalue, and x_i the ith eigenvector. Here we assume that $\lambda_1 < \lambda_2 < \ldots < \lambda_N$. Taking partial derivatives of Eqs. (1) and (2) with respect to a design parameter b yields the following equation for the eigenvector derivative $\partial x_i / \partial b$,

$$A_i\left(\frac{\partial x_i}{\partial b}\right) = F_i \tag{3}$$

$$\left(\frac{\partial x_i^T}{\partial b}\right)Mx_i + x_i^T M\left(\frac{\partial x_i}{\partial b}\right) = -x_i^T\left(\frac{\partial M}{\partial b}\right)x_i \tag{4}$$

where

$$A_i = K - \lambda_i M \tag{5}$$

$$F_i = -\left(\frac{\partial A_i}{\partial b}\right)x_i \tag{6}$$

and

$$\left(\frac{\partial A_i}{\partial b}\right) = \left(\frac{\partial K}{\partial b}\right) - \left(\frac{\partial \lambda_i}{\partial b}\right)M - \lambda_i\left(\frac{\partial M}{\partial b}\right) \tag{7}$$

The eigenvalue derivative is given by

$$\left(\frac{\partial \lambda_i}{\partial b}\right) = x_i^T\left[\left(\frac{\partial K}{\partial b}\right) - \lambda_i\left(\frac{\partial M}{\partial b}\right)\right]x_i \tag{8}$$

B. Exact Solution by Modal Superposition Method

For an N-degrees-of-freedom system, if all of the modes are available, the eigenvector derivative of $\partial x_i/\partial b$ can be exactly computed using its modal expansion representation[1]

$$\left(\frac{\partial x_i}{\partial b}\right) = C_0 x_i + X\gamma \tag{9}$$

where $X = [x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_N]$ is an $N \times (N-1)$ matrix C_0 a scale constant, and γ an $(N-1)$ column vector. C_0 and γ are given by

$$C_0 = -(1/2)x_i^T\left(\frac{\partial M}{\partial b}\right)x_i \tag{10}$$

$$\gamma = (\Lambda - \lambda_{0i} I)^{-1} X^T F_i \tag{11}$$

where $\Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{i-1}, \lambda_{i+1}, \ldots, \lambda_N)$.

C. New Accurate Modal Superposition Method

For a large structural system usually only a truncated set of low-frequency eigenvectors are available. Assume that L ($L < N$) low-frequency modes $x_1, x_2, \ldots, x_i, \ldots, x_L$ are available for the system with N degrees of freedom. In terms of the availability of vibration modes, Eq. (9) is rewritten as

$$\left(\frac{\partial x_i}{\partial b}\right) = C_0 x_i + X_{L-1}\gamma_{L-1} + S_h \tag{12}$$

$$S_h = X_h \gamma_h \tag{13}$$

where X_h is an $N \times (N-L)$ high-frequency modal matrix, $X_h = [x_{L+1}, \ldots, x_n]$, and γ_h is an $(N-L)$ column vector. X_{L-1} is an $N \times (N-L)$ low-frequency modal matrix, $X_{L-1} = [x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_L]$, and γ_{L-1} is an $(L-1)$ column vector. The values γ_{L-1} and γ_h are given by

$$\gamma_{L-1} = (\Lambda_{L-1} - \lambda_i I)^{-1} X_{L-1}^T F_i \tag{14}$$

$$\gamma_h = (\Lambda_h - \lambda_i I)^{-1} X_h^T F_i \tag{15}$$

where $\Lambda_h = \text{diag}(\lambda_{L+1}, \lambda_{L+2}, \ldots, \lambda_N)$ and $\Lambda_{L-1} = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{i-1}, \lambda_{i+1}, \ldots, \lambda_L)$.

As can be seen from Eqs. (13) and (14), S_h can not be directly obtained because the X_h is unavailable. Therefore, X_h is often neglected in practice. However, the errors due to neglecting S_h are significant when $L \ll N$. In this new accurate method, we seek an accurate approximation to S_h.

Using Eq. (15), S_h can be rewritten as

$$S_h = X_h(\Lambda_h - \lambda_i I)^{-1} X_h^T F_i \tag{16}$$

Since $\lambda_i < \lambda_{L+1}$, the matrix $(\Lambda - \lambda_i I)^{-1}$ can be expanded into a convergent series,

$$(\Lambda_h - \lambda_i I)^{-1} = \Lambda_h^{-1} + \lambda_i \Lambda_h^{-2} + \lambda_i^3 \Lambda_h^{-3} +, \ldots \tag{17}$$

It is well known that the series in Eq. (18) converges if $\lambda_i / \lambda_L < 1$.

Substituting Eq. (17) into Eq. (16), Eq. (16) becomes

$$S_h = \sum_{j=0}^{\infty} \lambda_i^j \{X_h \Lambda_h^{-j-1} X_h^T\} Fi \tag{18}$$

It can be shown that $X_h \Lambda^{-k} X_h^T$ can be expressed with available low-frequency modes and system matrices as [see Appendix],

$$X_h \Lambda_h^{-j-1} X_h^T = K^{-1}(MK^{-1})^j - X_L \Lambda_L^{-j-1} X_L^T \qquad (j = 0, 1, 2, \ldots) \tag{19}$$

where Λ_L is the available low-frequency eigenvalue diagonal matrix and X_L is the corresponding eigenvector matrix given by

$$\Lambda_L = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_L)$$

$$X_L = [x_1, x_2, \ldots, x_L]$$

Using Eqs. (18) and (19), the contribution due to unavailable high-frequency modes to the eigenvector derivatives can be expressed explicitly with the available low-frequency modes and system matrices.

Substituting Eq. (19) into Eq. (18) yields

$$S_h = \sum_{j=0}^{\infty} \lambda_i^j (H_j - W_j) \tag{20}$$

where

$$H_j = K^{-1}(MK^{-1})^j F_i \qquad (j = 0, 1, 2, \ldots) \tag{21a}$$

$$W_j = X_L \Lambda^{-j-1} X_L^T F_i \qquad (j = 0, 1, 2, \ldots) \tag{21b}$$

From Eq. (21) one can ascertain that W_j is easy to compute. H_j in Eq. (21) can be obtained with the following iterative procedure:

$$H_0 = K^{-1} F_i \tag{22}$$

$$H_{j+1} = K^{-1}(MH_j) \qquad (j = 0, 1, 2, \ldots) \tag{23}$$

Define $S_h(k)$ as

$$S_h(k) = \sum_{j=0}^{k} \lambda_i^j (H_j - W_j) \tag{24}$$

Using this definition, the given iterative process can be terminated if the next inequality

$$\|S_h(j) - S_h(j-1)\| < \varepsilon \tag{25}$$

is satisfied, where ε is some specified accuracy requirement.

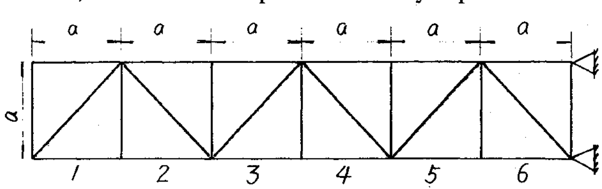

Fig. 1 Plane truss structure.

As can be seen from Eq. (12), the key point of this method is to add the term S_h to the eigenvector derivative to account for the contribution of the truncated modes. To compute S_h, S_h is exactly expressed as a convergent series shown in Eq. (20). Using the iterative process stated by Eqs. (22–24), this series can be computed easily and accurately. In fact, Ref. 4 also tries to give a method to compute S_h. Through comparison with Ref. 4, it can be seen that this new method is reduced to the explicit method[4] by Wang if only the first term of the series in Eq. (20) is retained with all of the other terms neglected. In addition, this series can be used to estimate the errors induced by the modal truncation.

III. Numerical Example

This numerical example is used to demonstrate and compare the solution accuracy for the three different methods including the modal truncation method that neglects S_h, Wang's explicit method proposed in Ref. 4 and the present method.

For the sake of simplicity, the error of the eigenvector derivative is defined as $\|(\partial x_i/\partial b_j)_e - (\partial x_i/\partial b_j)_a\|$, where $(\partial x_i/\partial b_j)_e$ represents the exact solution for the ith eigenvector derivative with respect to the jth design parameter, $(\partial x_i/\partial b_j)_a$ represents the approximate solution corresponding to $(\partial x_i/\partial b_j)_e$. Here, the Nelson method[3] is used to compute $(\partial x_i/\partial b_j)_e$.

Consider a plane-truss structure shown in Fig. 1 with the parameters given by $a = 100$ cm, $b = 6$ cm^2, $\rho = 7.8$ g/cm^3, $E = 2.1 \times 10^{12}$ g/cm · s, and $\mu = 0.3$, where b represents the homogeneous cross-sectional area of each rod, ρ the mass density, E Young's modulus, and μ the Possion ratio.

Now select the cross-sectional area of certain rods to be a design parameter. Two cases are considered. One is to treat the cross-sectional area of the six rods, rods 1–6, as a design parameter, denoted by b_1; the other is to treat the cross-sectional area of only one rod, rod 4, as a design parameter, denoted by b_2. The derivatives of the first two eigenvectors with respect to these two parameters are computed, respectively, using the aforementioned methods. In each method, the number of modes in modal superposition and/or the number of series terms retained in S_h varies for the purpose of comparison. The error of the aforementioned methods is included in Table 1.

The numerical values in Table 1 confirm the following.

1) The error of modal truncation method is sometimes significant if only few lower frequency modes are retained although a large number of higher frequency modes are truncated. It can be seen from the first column in Table 1 that the errors associated with the modal truncation method are relatively large ranging from 6.67 to 93.82%.

The reason is that eigenvector derivatives are usually more local than the eigenvector of concern. Therefore, the eigenvector derivatives may not be dominated by the first modes, which are generally global.

2) When the same number of the first modes are used in superposition, the present method is the most accurate of all of the three methods, and the Wang method is more accurate than the modal truncation method. It can be seen from each row in Table 1 that the error of the present method is the smallest, whereas the error of the modal truncation method is the largest. For instance, when using two modes, the error of $\partial x_2/\partial b_2$ by the modal truncation method is 93.82%, that by the Wang method is reduced to 41.97%, and that by the present method is further reduced to 20.43% and 10.06%, respectively.

3) The present method is more accurate than the modal truncation method even if the total sum of modes used and series terms retained in the present method is equal to the number of modes used in the modal truncation method. Taking $\partial x_1/\partial b_2$ as an example, when four modes are used in the modal truncation method, its error is 26.29%, whereas two modes and two series terms are used in the present method, the error is reduced to 0.00%

IV. Concluding Remarks

An accurate method for computing eigenvector derivatives in structural dynamics is presented in this Note. The new idea in this Note is that the contribution due to the unavailable high-frequency modes is given by a convergent series which can be computed without using the truncated modes. This series not only improves the solution accuracy but also provides the error estimates of the computed results. In addition, if the first terms in this series are used as the basis vectors in Wang's implicit method[4] to approximately span a space of the eigenvector derivative of concern, the accuracy of Wang's implicit method can be further improved. In this new method the extra computational cost is to solve a set of static response linear equations with a fixed coefficient matrix and different right sides.

Table 1 Errors of eigenvector derivatives

	Number of modes used	Modal truncation method, %	Wang's method, %	Present method, % Number of series terms 2	3
Error of $\partial x_1/\partial b_1$	2	51.08	1.41	0.03	0.00
	3	6.67	0.03	0.00	0.00
	4	6.67	0.03	0.00	0.00
Error of $\partial x_2/\partial b_1$	2	90.80	39.94	19.27	9.46
	3	41.64	8.31	1.92	0.48
	4	24.83	2.27	0.25	0.13
Error of $\partial x_1/\partial b_2$	2	49.47	1.07	0.02	0.00
	3	29.50	0.23	0.00	0.00
	4	26.29	0.12	0.00	0.00
Error of $\partial x_2/\partial b_2$	2	93.82	41.97	20.43	10.06
	3	37.95	6.65	1.53	0.42
	4	28.54	2.32	0.27	0.21

Appendix

From eigenproblems (1) and (2), it can be shown that

$$[X_L \mid X_h]^T K [X_L \mid X_h] = \mathrm{diag}(\Lambda_L \mid \Lambda_h) \tag{A1}$$

where X_L is the low-frequency eigenvector matrix, Λ_L the corresponding low-frequency eigenvalue diagonal matrix, X_h the high-frequency eigenvector matrix, and Λ_h the corresponding high-frequency diagonal matrix.

Inversion of the two sides of Eq. (A1) yields

$$K^{-1} = [X_L \mid X_h]\, \mathrm{diag}(\Lambda_L^{-1} \mid \Lambda_h^{-1})\, [X_L \mid X_h]^T \tag{A2}$$

From Eq. (A2), it can be shown that

$$X_h \Lambda_h^{-1} X_h^T = K^{-1} - X_L \Lambda_L^{-1} X_h^T \tag{A3}$$

Postmultiplying the two sides of Eq. (A2) by MK^{-1} yields

$$K^{-1}MK^{-1} = [X_L \mid X_h]\, \mathrm{diag}(\Lambda_L^{-1} \mid \Lambda_h^{-1})\, [X_L \mid X_h]^T MK^{-1} \tag{A4}$$

Introducing Eq. (A2) into the right side of Eq. (A4), Eq. (A4) becomes

$$K^{-1}MK^{-1} = [X_L \mid X_h]\ \mathrm{diag}(\Lambda_L^{-2} \mid \Lambda_h^{-2})\, [X_L \mid X_h]^T \tag{A5}$$

Equation (A5) can be rewritten as

$$X_h \Lambda_h^{-2} X_h^T = K^{-1}(MK^{-1}) - X_L \Lambda_L^{-2} X_L^T \tag{A6}$$

In a similar way, it can be shown that

$$X_h \Lambda_h^{-j-1} X_h^T = K^{-1}(MK^{-1})^j - X_L \Lambda_L^{-j-1} X_L^T \qquad (j = 0, 1, 2, \ldots) \tag{A7}$$

Acknowledgment

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